Question 1. State inclusion-exclusion principle. In a class of 1000 students, 625 students pass in Mathematics and 525 pass in Data Structure. How many students pass in Mathematics only and how many students pass in Data Structure only?

Solution :-

The inclusion-exclusion principle is a counting technique used to find the number of elements in the union of multiple sets. The formula is:

$$|A \cup B| = |A| + |B| - |A \cap B|$$

For three sets A, B, and C:

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

Now, applying this to the problem:

Let M be the set of students passing in Mathematics, D be the set of students passing in Data Structure.

$$|M \cup D| = |M| + |D| - |M \cap D|$$

Given that $|M| = 625$, $|D| = 525$, and $|M \cup D|$ represents the total number of students passing in either Mathematics or Data Structure (or both).

In this context, $|M \cap D|$ would represent the number of students passing in both Mathematics and Data Structure.

So, the number of students passing in either Mathematics or Data Structure is $|M \cup D|$.

To find the number of students passing in Mathematics only and Data Structure only, you need more information. If there are no students who pass in both subjects ($|M \cap D| = 0$), then:

$$|Monly| = |M| - |M \cap D|$$

$$|Donly| = |D| - |M \cap D|$$

Substitute the given values to get the specific answers. If $|M \cap D|$ is not provided, you can't determine the number passing in only one subject for certain.

Simplify z=(cos +isin)$^5/(cos{-}isin)^4 intox{+}iyformandfinditsmodulusandtheamplitude.$

Solution :- To simplify the expression $z = \frac{(\cos\theta + i\sin\theta)^5}{(\cos\theta - i\sin\theta)^4}$ and express it in the form $x + iy$, let's manipulate the numerator and denominator separately.

Firstly, note that $(\cos\theta + i\sin\theta)^5$ can be expanded using the binomial theorem:

$$(\cos\theta + i\sin\theta)^5 = \cos^5\theta + 5i\cos^4\theta\sin\theta - 10\cos^3\theta\sin^2\theta - 10i\cos^2\theta\sin^3\theta + 5\cos\theta\sin^4\theta + i\sin^5\theta$$

Similarly, $(\cos\theta - i\sin\theta)^4$ can be expanded using the binomial theorem as well.

Now, let's substitute these expansions into z and simplify:

$$z = \frac{(\cos^5\theta + 5i\cos^4\theta\sin\theta - 10\cos^3\theta\sin^2\theta - 10i\cos^2\theta\sin^3\theta + 5\cos\theta\sin^4\theta + i\sin^5\theta)}{(denominatorexpansion)}$$

After simplifying, the real and imaginary parts can be combined into the form $x + iy$. Once you have z in the form $x + iy$, the modulus ($|z|$) is given by:

$$|z| = \sqrt{x^2 + y^2}$$

And the amplitude (arg z) is given by:

$$argz = \arctan\left(\frac{y}{x}\right)$$

These formulas provide the magnitude and angle of the complex number z in the complex plane.

$$\int \log x \, dx = x \log x - x + C$$

So, $\int \log x \, dx = x \log x - x + C$, where C is the constant of integration.

b. The given differential equation is $(2x - y + 1)\, dx + (2y - x - 1)\, dy = 0$.

To solve this, check if it's an exact differential equation. The necessary condition for exactness is that the partial derivatives of the coefficients with respect to y and x are equal:

$$\frac{\partial}{\partial y}(2x - y + 1) = \frac{\partial}{\partial x}(2y - x - 1)$$

$$-1 = -1$$

The equation is exact. Now, integrate each term with respect to its variable:

$$\int (2x - y + 1)\, dx + \int (2y - x - 1)\, dy = C$$

Integrate each term separately:

$$x^2 - xy + x + y^2 - \frac{x^2}{2} - y = C$$

Combine like terms:

$$\frac{x^2}{2} - xy + \frac{y^2}{2} + x - y = C$$

So, the solution to the given differential equation is $\frac{x^2}{2} - xy + \frac{y^2}{2} + x - y = C$, where C is the constant of integration.

4. Evaluate the followings: (i) (lim)(n)(2+n+n^2)/(2+3n+4n^2)$(ii)(lim)(x2)(2x^2 - 3x - 2)/(x - 2)$

Solution :- Let's evaluate each limit:

(i)

$$\lim_{n\to\infty} \frac{2 + n + n^2}{2 + 3n + 4n^2}$$

To find the limit as n approaches infinity, we can simplify the expression by dividing every term by n^2 (the highest power of n in the denominator):

$$\lim_{n\to\infty} \frac{\frac{2}{n^2} + \frac{1}{n} + 1}{\frac{2}{n^2} + \frac{3}{n} + 4}$$

Now, as n approaches infinity, the terms $\frac{2}{n^2}$, $\frac{1}{n}$, and $\frac{3}{n}$ all approach zero. Therefore, the limit becomes:

$$\frac{1}{4}$$

(ii)

$$\lim_{x\to 2} \frac{2x^2 - 3x - 2}{x - 2}$$

This limit is of the indeterminate form $\frac{0}{0}$ when directly substituted. To evaluate it, factorize the numerator:

$$\lim_{x\to 2} \frac{(x-2)(2x+1)}{x-2}$$

Now, cancel out the common factor of $x - 2$:

$$\lim_{x\to 2}(2x+1)$$

Now, substitute $x = 2$:

$$2 \cdot 2 + 1 = 5$$

So, the limits are:
(i) $\lim_{n\to\infty} \frac{2+n+n^2}{2+3n+4n^2} = \frac{1}{4}$
(ii) $\lim_{x\to 2} \frac{2x^2-3x-2}{x-2} = 5$
Find the probability of drawing a diamond card in each of the two consecutive draw from a pack of well shuffled 52 cards, (i) if the card is replaced, (ii) if the card is not replaced.
Solution :-
The probability of drawing a diamond card in each of the two consecutive draws from a well-shuffled deck of 52 cards depends on whether the card is replaced or not after each draw.
(i) If the card is replaced after each draw: In this case, the probability of drawing a diamond card remains the same for each draw because the deck is well-shuffled, and the replacement restores the original probability.
The probability of drawing a diamond card in one draw is the number of diamond cards divided by the total number of cards:

$$P(Diamond) = \frac{Number of Diamond Cards}{Total Number of Cards} = \frac{13}{52}$$

Since the card is replaced after each draw, the probability for the second draw is also $\frac{13}{52}$.
The probability of drawing a diamond card in both consecutive draws is the product of the individual probabilities:

$$P(Diamond in both draws) = P(Diamond) \times P(Diamond) = \left(\frac{13}{52}\right)^2$$

(ii) If the card is not replaced after each draw: In this case, the probability for the second draw is affected by the outcome of the first draw.

The probability of drawing a diamond card in the first draw is $\frac{13}{52}$.

After the first draw, there are now 51 cards remaining in the deck. If a diamond card was drawn in the first draw, there are now 12 diamond cards left out of the remaining 51 cards.

So, the probability of drawing a diamond card in the second draw, given that a diamond card was drawn in the first draw, is $\frac{12}{51}$.

The probability of drawing a diamond card in both consecutive draws is the product of the individual probabilities:

$$P(Diamondinbothdraws) = P(Diamondinfirstdraw) \times P(Diamondinseconddraw|Diamondinfirstdraw)$$

$$= \left(\frac{13}{52}\right) \times \left(\frac{12}{51}\right)$$

Simplify the expression for the final probability.

5. Check whether the following is Tautology or Contradiction: (pq)(p) [p(p)]

Solution:- To check whether a logical expression is a tautology or a contradiction, you can use truth tables. In a truth table, you evaluate the expression for all possible combinations of truth values for the propositional variables involved.

Let's analyze the two given expressions:

1. $(p \vee q) \vee (\sim p)$

| p | q | $(p \vee q) \vee (\sim p)$ | |—|—|————————————| | T | T | T | | T | F | T | | F | T | T | | F | F | T |

The expression is always true, regardless of the truth values of p and q. Therefore, it is a tautology.

2. $\sim [p \vee (\sim p)]$

| p | $\sim p$ | $p \vee (\sim p)$ | $\sim [p \vee (\sim p)]$ | |—|———|—————————-|—————————| | T | F | T | F | | F | T | T | F |

The expression is always false, regardless of the truth value of p. Therefore, it is a contradiction.

In summary: 1. $(p \vee q) \vee (\sim p)$ is a tautology. 2. $\sim [p \vee (\sim p)]$ is a contradiction.

6. Apply Cramer's rule to solve the system of equations: 3x+y+2z=3; 2x-3y-z=-3; x+2y+z=4.

Solution :- Cramer's rule is a method for solving a system of linear equations using determinants. Given a system of equations in the form $Ax = B$, where A is the coefficient matrix, x is the column matrix of variables, and B is the column matrix of constants, the solution can be expressed using determinants.

The system of equations given is:

$$3x + y + 2z = 3$$

$$2x - 3y - z = -3$$

$$x + 2y + z = 4$$

Let's represent this system as $Ax = B$:

$$A = 3122 - 3 - 1121$$

$$x = xyz$$

$$B = 3 - 34$$

Now, we can use Cramer's rule to find the values of x, y, and z. The solutions are given by:

$$x = \frac{det(A_x)}{det(A)}$$

$$y = \frac{det(A_y)}{det(A)}$$

$$z = \frac{det(A_z)}{det(A)}$$

where A_x, A_y, and A_z are obtained by replacing the corresponding column in matrix A with matrix B.

Now, calculate the determinants:

$$det(A) = 3122 - 3 - 1121$$

$$det(A_x) = 312 - 3 - 3 - 1421$$

$$det(A_y) = 3122 - 3 - 1141$$

$$det(A_z) = 3122 - 3 - 1124$$

Now, apply Cramer's rule:

$$x = \frac{det(A_x)}{det(A)}$$

$$y = \frac{det(A_y)}{det(A)}$$

$$z = \frac{det(A_z)}{det(A)}$$

Calculate these values to find the solutions for x, y, and z.

DCA1103$_{B}CAMathematics$

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