Sage and Linear Algebra Worksheet

FCLA Section IVLT

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1 Invertible Linear Transformations

A carefully-crafted invertible linear transformation from $\mathbb{Q}^5$ to $\mathbb{Q}^5$.

```
A = matrix(QQ, [[1, 1, -1, -2, 0], [-3, -2, 1, 4, 7],
    [2, 2, -1, -3, -4], [-4, -3, 3, 8, 3], [5, 6, -7, -8,
        8]])
T = linear_transformation(QQ^5, QQ^5, A, side='right')
T
```

```
T.is_injective(), T.is_surjective()
```

```
T.is_invertible()
```

```
S = T.inverse()
S
```

The `*` operator, like we would use for multiplication, will create a composition. This will be perfectly natural once we discuss Section MR. Here, composing an invertible linear transformation with its inverse will yield an identity linear transformation.

```
comp = S*T
comp
```

```
comp.is_identity()
```

2 Defining an Invertible Linear Transformation on Bases

Now, an invertible linear transformation defined on a basis, and the resulting inverse linear transformation. We get two "random" bases of $\mathbb{Q}^7$ from nonsingular (determinant one) matrices.

```
C = random_matrix(QQ, 7, 7, algorithm='unimodular',
    upper_bound=99)
```

```
Cbasis = C.columns()
D = random_matrix(QQ, 7, 7, algorithm='unimodular',
    upper_bound=99)
Dbasis = D.columns()
```

Vector spaces with defined user bases.

```
Cspace = (QQ^7).subspace_with_basis(Cbasis)
Dspace = (QQ^7).subspace_with_basis(Dbasis)
Cspace, Dspace
```

The invertible linear transformation defined with images as the vectors in the codomain basis `D`.

```
T = linear_transformation(Cspace, QQ^7, Dbasis)
T
```

```
T.is_invertible()
```

Now we simply “turn around” the definition, to make the inverse.

```
S = linear_transformation(Dspace, QQ^7, Cbasis)
S
```

```
S.is_invertible()
```

Composition with vector spaces using different bases does not seem to be working properly. So we just check some random inputs to the composition.

```
comp = S*T
comp.is_identity()
```

```
v = random_vector(QQ, 7)
v, T(S(v)) == v, S(T(v)) == v
```

3 Rank and Nullity

A general (i.e. not invertible) linear transformation from $\mathbb{Q}^6$ to $\mathbb{Q}^5$.

```
F = matrix(QQ, [[1, 0, 2, -1, -4, 2], [-1, -1, -4, 3, 6,
    -5], [0, 1, 3, -2, -4, 5],
    [0, 4, 6, -8, -4, 8], [0, 1, 2, -2, -2, 3]])
R = linear_transformation(QQ^6, QQ^5, F, side='right')
R
```

Rank is dimension of range (image). Note there are not left/right variants.

```
R.image()
```

```
R.rank()
```

Nullity is dimension of kernel. Note there are not left/right variants.

```
R.kernel()
```

```
R.nullity()
```

Note that rank and nullity sum to the dimension of the domain (which is 6 here).