Hooked on Calculus: Crochet Surfaces of Revolution Worksheet

Choose a function $f(x)$ and horizontal bounds $a \leq x \leq b$ on which $f(x)$ is differentiable and positive valued. Generate a crochet pattern for the corresponding surface of revolution. You may use the calculator at https://www.desmos.com/calculator/8rfmnuciwg for this worksheet. Enter your own function $f(x)$ and your own bounds a and b at the top.

1. For your chosen function and bounds, calculate the arc length using the formula
$$\int_a^b \sqrt{1 + (f'(x))^2}\, dx.$$
2. Compare the previous number with the number of rows in your pattern. Are they similar? Explain.
3. Consider
$$\int_a^{x_1} \sqrt{1 + (f'(x))^2}\, dx.$$
Find an $x_1 > a$ such that the integral approximately equals 0.5.
4. Calculate $2\pi f(x_1)$ and compare this with the number of chain stitches in Row 1. Explain.
5. Consider
$$\int_a^{x_2} \sqrt{1 + (f'(x))^2}\, dx.$$
Find an $x_2 > x_1$ such that the integral approximately equals 1.5.
6. Calculate $2\pi f(x_2)$ and compare this with the number of stitches in Row 2. Explain.
7. Using the above exploration as guidance, what are the main mathematical concepts behind our pattern creation for surfaces of revolution and why?
8. For your chosen function, calculate the surface area of the surface of revolution using the formula
$$\int_a^b 2\pi f(x)\sqrt{1 + (f'(x))^2}\, dx.$$
9. Compare the previous number with the number of stitches in your pattern. Are they similar? Explain.
10. For your chosen function, calculate the volume inside the surface of revolution using the formula
$$\int_a^b \pi (f(x))^2\, dx.$$
11. Include a printout of your program worksheet that shows your function, bounds, and the pattern generated.
12. Share your completed crochet project with Dr. Taylor, preferably in person, but pictures are also welcome!